Graduate course

Geometric Data Analysis (GDA)

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Table of Contents

1. Introduction to GDA methods
   - The Three Key Ideas of GDA
   - Three Paradigms
   - Frame model
   - Historical Sketch

2. Principal Axes of a Euclidean Cloud
   - Basic Geometric Notions
   - Cloud of Points
   - Principal Axes of a Cloud
   - From a Plane Cloud to a Higher Dimensional Cloud
   - Properties and Aids to Interpretation
   - Results of the Analysis

3. Multiple Correspondence Analysis
   - Principles of MCA
   - Taste example
   - Construction of the two Clouds
   - Principal Clouds
Table of Contents II

- Aids to Interpretation: Contributions to axes
- MCA of the Taste Example
- Interpretation of the Analysis of the Taste Example
- Transition Formulas
- Supplementary Variables
- Specific MCA
- Study of a Subcloud
- Class Specific Analysis (CSA)

4 IV – Structured Data Analysis

5 V – Euclidean Clustering
- Introduction
- Partition of a Cloud: Between– and Within–variance
- $K$–means Clustering
- Ascending Hierarchical Clustering (AHC)
- AHC with Euclidean distance
- Interpretation of clusters
- Other Aggregation Indices
VI – Combinatorial Inference in GDA

- Introduction
- Typicality Tests
- Hypergeometric Typicality Test for a Relative Frequency
- Homogeneity test
I – Introduction to Geometric Data Analysis methods
I.1. The Three Key Ideas of GDA

1. Geometric modeling

- Data table
- Clouds

<table>
<thead>
<tr>
<th>Individuals</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>a1</td>
</tr>
<tr>
<td></td>
<td>b1</td>
</tr>
<tr>
<td></td>
<td>c3</td>
</tr>
</tbody>
</table>

Cloud of categories: Points represent the categories of variables.

Cloud of individuals: Points represent individuals.

2. Formal approach.

3. Inductive philosophy

The model should follow the data, not the reverse!
I.2. Three Paradigms

- **Correspondence Analysis (CA)**
  - $\rightarrow$ Contingency table

- **Principal Component Analysis (PCA)**
  - $\rightarrow$ Individuals $\times$ Numerical Variables table

- **Multiple Correspondence Analysis (MCA)**
  - $\rightarrow$ Individuals $\times$ Categorical Variables table
I.3. Frame model

In Geometric Data Analysis, two principles should be followed (Benzécri, 1992, pp. 382-383):

- Homogeneity
- Exhaustiveness

Benzécri, 1992, pp. 382-383
1.4. Historical Sketch

Precursors

Karl Pearson (1901), Hirschfeld (1935).

Should we need an Anglo–Saxon patronage for “Analyse des Données”, we would be pleased to turn to the great Karl Pearson.

Benzécri (1982), p. 116

- Optimal scaling: Fisher (1940), Guttman (1942)
- Factor analysis: Burt (1950)
- Quantification method: Hayashi (1952)
Emergence (1963-73)

Benzécri et al. (1973): L’ANALYSE Des DONNÉES
1 la TAXINOMIE 2 L’ANALYSE DES CORRESPONDANCES
Recognition and splendid isolation (1973-1980)

Gower (1966), Good (1969), Gabriel (1971)
Ignored in Shepard, Romney, Nerlove (1972), Kruskal & Wish (1978), Shepard (1980) and in Kendall & Stuart (1976)
International recognition (since 1981)

Benzécri (1992) (translation of the introductory book published by Dunod in 1984);

“Econometrics without stochastic models”

Tenenhaus & Young (1985): Psychometry;
Nishisato (1980): Dual Scaling;
Carroll & Green (1988), Weller & Romney (1990): MDS group;
Where do we stand now?

**CARME network:**

**Workshops** organized in Paris, Uppsala, Copenhagen, Montreux, London, Kaliningrad, Mendoza, Berkeley, Potsdam . . .
Recent Books:

Le Roux & Rouanet

Murtagh 2005

CARME 2003 (2006)
CA is now recognized and used, but GDA as a whole methodology, is beginning to be discovered by a large audience.
This text is adapted from Chapter 2 of the monograph

*Multiple Correspondence Analysis*

(QASS series n°163, SAGE, 2010)
II.1. Basic Geometric Notions

Elements of a geometric space: *points, line, plane.*

— *Affine notions:* alignment, direction and barycenter.

Couple of points *(P, M)*, or *dipole* \( \rightarrow \text{vector} \ \overrightarrow{PM} \)

The *deviation* from point *P* to point *M* is \( M - P \) ("terminal minus initial"), that is, \( \overrightarrow{PM} \).

Deviations add up vectorially: sum of vectors by *parallelogram law*

\[
\overrightarrow{PM} + \overrightarrow{PN} = \overrightarrow{PQ}
\]
Barycenter of a dipole

\[
B \quad (b = 2) \\
G \\
A \quad (a = 3)
\]

\[
G = \frac{3A + 2B}{5}
\]

\[
\overrightarrow{PG} = \frac{3}{5} \overrightarrow{PA} + \frac{2}{5} \overrightarrow{PB}
\]

Barycenter = weighted average of points: \( G = \frac{aA + bB}{a + b} \)
Metric notions: distances and angles.

Triangle inequality:
\[ PQ \leq PM + MQ \]

Pythagorean theorem:
If PM and MQ are perpendicular then:
\[ (PM)^2 + (MQ)^2 = (PQ)^2 \]
(triangle MPQ with right angle at M),
II.2. Cloud of Points

Figure 1. Target example (10 points)
Figure 1b. Target example with origine-point $O$ and initial axes

Initial coordinates

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i1$</td>
<td>0</td>
<td>-12</td>
<td>1</td>
</tr>
<tr>
<td>$i2$</td>
<td>6</td>
<td>-10</td>
<td>1</td>
</tr>
<tr>
<td>$i3$</td>
<td>14</td>
<td>-6</td>
<td>1</td>
</tr>
<tr>
<td>$i4$</td>
<td>6</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>$i5$</td>
<td>12</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$i6$</td>
<td>-8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$i7$</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$i8$</td>
<td>6</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$i9$</td>
<td>10</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>$i10$</td>
<td>12</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Means 6 0
Variances 40 52
Covariance +8
Mean Point

Cloud of points \( M^i \) with relative weights \( p_i \).

Mean point: point \( G \)

\[
\overrightarrow{OG} = \sum p_i \overrightarrow{OM^i} \quad \sum p_i \overrightarrow{GM^i} = \overrightarrow{0} \quad \text{(barycentric property)}
\]

Target Example: \( p_i = \frac{1}{10} \)
Variance, contribution

Variance of a cloud:

\[ V_{\text{cloud}} = \sum p_i (GM_i)^2 \]

Property

In rectangular axes, the variance of the cloud is the sum of the variances of the coordinate variables.

Contribution of point \( M^i \):

\[ Ctr_i = \frac{p_i(GM_i)^2}{V_{\text{cloud}}} \]
II.3. Principal Axes of a Cloud

*Projection of a cloud*

\[ P' = \text{projection of point } P \text{ onto } \mathcal{L} \text{ along } \mathcal{L}' \]

\[ \overrightarrow{P'P} = \text{residual deviation} \]
If point $M$ is the midpoint of $P$ and $Q$, the point $M'$, projection of $M$ on $\mathcal{L}$, is the midpoint of $P'$ and $Q'$.

**Mean point property**

The mean point is preserved by projection.
**Orthogonal projection:** $PP'$ is perpendicular to $\mathcal{L}$.

The orthogonal projection contracts distances: $P'Q' \leq PQ$, therefore one has the

**Property**

\[ \text{variance of projected cloud } \leq \text{variance of initial cloud}. \]
Projected clouds on several lines

Orthogonal additive decomposition

The variance of the initial cloud is the sum of the variances of projected clouds onto perpendicular lines: $V_{cloud} = 40 + 52 = 92$. 

\[ \text{variance} = 40 \]

\[ \text{variance} = 52 \]
Projection onto an oblique line (60 degrees): variance = 55.9
II – Principal Axes of a Euclidean Cloud

Principal Axes of a Cloud

$\mathcal{D}_1$ $\mathcal{D}_2$ $\mathcal{D}_3$ $\mathcal{D}_4$ $\mathcal{D}_5$ $\mathcal{D}_6$

Variance

<table>
<thead>
<tr>
<th>$\mathcal{D}_1$</th>
<th>$\mathcal{D}_2$</th>
<th>$\mathcal{D}_3$</th>
<th>$\mathcal{D}_4$</th>
<th>$\mathcal{D}_5$</th>
<th>$\mathcal{D}_6$</th>
<th>$\mathcal{D}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>42.1</td>
<td>36.1</td>
<td>40.0</td>
<td>49.9</td>
<td>55.9</td>
<td>52</td>
</tr>
</tbody>
</table>

angle in degrees
The line whose the variance of the projected cloud is maximum is called \textit{first principal line}. directed line $\rightarrow$ \textit{1st principal axis}

Projected cloud = \textit{1st principal cloud}
its variance ($\lambda_1$) = \textit{variance of axis 1}

The first principal cloud is the best fitting of the initial cloud by an unidimensional cloud in the sense of \textit{orthogonal least squares}

Here, angle $= 63^\circ$, $\lambda_1 = 56$. 

\[ \text{axis 1} \]
\[ \lambda_1 = 56 \]

\[
\begin{align*}
\text{i1} & \quad \text{i2} & \quad \text{i6} & \quad \text{i4} & \quad \text{i7} & \quad \text{i8} & \quad \text{i9} & \quad \text{i10} \\
\text{i3} \quad \text{G} & \quad i5 & \quad i10 & \quad i9 & \quad i8 & \quad i7 & \quad i1 & \quad i2 & \quad i6
\end{align*}
\]
The residual cloud is constructed as the orthogonal projection of the cloud on the subspace orthogonal to the first principal line.

The first principal line of the residual cloud defines the second principal line of the initial cloud.
II.4. From Plane Cloud to High Dimensional Cloud

Heredity property

The plane that best fits the cloud is the one determined by the first two axes.
II.5. Properties

- **Variance of cloud** = sum of variances of axes: \( V_{\text{cloud}} = \sum \lambda \ell \).

- The *principal axes* are *pairwise orthogonal*. Each axis can be directed arbitrarily.

- The *principal coordinates* of points define *principal variables*.
  
  - they have mean = 0 and variance = \( \lambda \) (eigenvalue)
  - they are *uncorrelated* (for distinct eigenvalues).
Aids to Interpretation

- Quality of fit of an axis or *variance rate*:
  \[ \frac{\lambda}{V_{\text{cloud}}} \]

- Contribution of point to axis:
  \[ Ctr = \frac{p(y)^2}{\lambda} \quad (p = \text{relative weight, } y = \text{coordinate on axis}) \]

- Quality of representation of point onto axis:
  \[ \cos^2 \theta = \frac{GP^2}{GM^2} \]
Results of the Analysis

\( \lambda_1 = 56 \) (variance of axis 1, eigenvalue), \( \lambda_2 = 36 \).

Variance rate: \( \frac{\lambda_1}{V_{\text{cloud}}} = \frac{56}{92} = 61\% \)

Principal representation of the cloud.
### Results for axis 1 ($\lambda_1 = 56$)

<table>
<thead>
<tr>
<th>$i$</th>
<th>Coordinates</th>
<th>Ctr (%)</th>
<th>squared cosines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_1$</td>
<td>0.1</td>
<td>$-13.41$</td>
<td>32.1</td>
</tr>
<tr>
<td>$i_2$</td>
<td>0.1</td>
<td>$-8.94$</td>
<td>14.3</td>
</tr>
<tr>
<td>$i_3$</td>
<td>0.1</td>
<td>$-1.79$</td>
<td>0.6</td>
</tr>
<tr>
<td>$i_4$</td>
<td>0.1</td>
<td>$-1.79$</td>
<td>1.3</td>
</tr>
<tr>
<td>$i_5$</td>
<td>0.1</td>
<td>$+2.68$</td>
<td>3.6</td>
</tr>
<tr>
<td>$i_6$</td>
<td>0.1</td>
<td>$-4.47$</td>
<td>3.6</td>
</tr>
<tr>
<td>$i_7$</td>
<td>0.1</td>
<td>$+1.79$</td>
<td>0.6</td>
</tr>
<tr>
<td>$i_8$</td>
<td>0.1</td>
<td>$+3.58$</td>
<td>2.3</td>
</tr>
<tr>
<td>$i_9$</td>
<td>0.1</td>
<td>$+10.73$</td>
<td>20.6</td>
</tr>
<tr>
<td>$i_{10}$</td>
<td>0.1</td>
<td>$+11.63$</td>
<td>24.1</td>
</tr>
</tbody>
</table>

### Results for axis 2 ($\lambda_2 = 36$)

<table>
<thead>
<tr>
<th>Coordinates</th>
<th>Ctr (%)</th>
<th>squared cosines</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>$+4.47$</td>
<td>5.6</td>
<td>0.20</td>
</tr>
<tr>
<td>$+9.84$</td>
<td>26.9</td>
<td>0.97</td>
</tr>
<tr>
<td>$+0.89$</td>
<td>0.2</td>
<td>0.20</td>
</tr>
<tr>
<td>$+5.37$</td>
<td>8</td>
<td>0.80</td>
</tr>
<tr>
<td>$-13.42$</td>
<td>50.0</td>
<td>0.90</td>
</tr>
<tr>
<td>$-5.37$</td>
<td>8</td>
<td>0.90</td>
</tr>
<tr>
<td>$-1.79$</td>
<td>0.9</td>
<td>0.20</td>
</tr>
<tr>
<td>$-0.89$</td>
<td>0.2</td>
<td>0.01</td>
</tr>
<tr>
<td>$+0.89$</td>
<td>0.2</td>
<td>0.01</td>
</tr>
</tbody>
</table>

- **Reconstitution of distances** between points:
  \[
  (M_i^1M_i^2)^2 = (-13.41 + 8.94)^2 + (0 - 4.47)^2 = 4.23 = (6.3)^2
  
  (GM_i^2)^2 = (-8.94)^2 + (-4.47)^2 = 100
  
- **Quality of representation** of point $M_i^2$: $\cos^2 \theta = \frac{(-8.94)^2}{100} = 0.80$
This text is adapted from Chapter 3 of the monograph

Multiple Correspondence Analysis
(QASS series n°163, SAGE, 2010)
III.1. Introduction

Language of questionnaire

Basic data set: Individuals × Questions table

- Questions = categorical variables, i.e. variables with a finite number of response categories (or modalities).
- Individuals or “statistical individuals”: (people, firms, items, etc.).

“Standard format”

for each question, each individual chooses one and only one response category.
→ otherwise: preliminary phase of coding
Table analyzed by mca: $I \times Q$ table

MCA produces two clouds of points:
the *cloud of individuals* and the *cloud of categories*. 
III.3. Taste example

• Data

$Q = 4$ active variables

Which, if any, of these different types of ... television programmes do you like the most?

<table>
<thead>
<tr>
<th>TV Programme</th>
<th>$n_k$</th>
<th>$f_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>News/Current affairs</td>
<td>220</td>
<td>18.1</td>
</tr>
<tr>
<td>Comedy/sitcoms</td>
<td>152</td>
<td>12.5</td>
</tr>
<tr>
<td>Police/detective</td>
<td>82</td>
<td>6.7</td>
</tr>
<tr>
<td>Nature/History documentaries</td>
<td>159</td>
<td>13.1</td>
</tr>
<tr>
<td>Sport</td>
<td>136</td>
<td>11.2</td>
</tr>
<tr>
<td>Film</td>
<td>117</td>
<td>9.6</td>
</tr>
<tr>
<td>Drama</td>
<td>134</td>
<td>11.0</td>
</tr>
<tr>
<td>Soap operas</td>
<td>215</td>
<td>17.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1215</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>
Which, if any, of these different types of … (cinema or television) films do you like the most? $n_k$ $f_k$ in %

<table>
<thead>
<tr>
<th>Genre</th>
<th>$n_k$</th>
<th>$f_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action/Adventure/Thriller</td>
<td>389</td>
<td>32.0</td>
</tr>
<tr>
<td>Comedy</td>
<td>235</td>
<td>19.3</td>
</tr>
<tr>
<td>Costume Drama/Literary adaptation</td>
<td>140</td>
<td>11.5</td>
</tr>
<tr>
<td>Documentary</td>
<td>100</td>
<td>8.2</td>
</tr>
<tr>
<td>Horror</td>
<td>62</td>
<td>5.1</td>
</tr>
<tr>
<td>Musical</td>
<td>87</td>
<td>7.2</td>
</tr>
<tr>
<td>Romance</td>
<td>101</td>
<td>8.3</td>
</tr>
<tr>
<td>SciFi</td>
<td>101</td>
<td>8.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1215</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>
Which, if any, of these different types of art do you like the most?

<table>
<thead>
<tr>
<th>Art</th>
<th>( n_k )</th>
<th>( f_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance Art</td>
<td>105</td>
<td>8.6</td>
</tr>
<tr>
<td>Landscape</td>
<td>632</td>
<td>52.0</td>
</tr>
<tr>
<td>Renaissance Art</td>
<td>55</td>
<td>4.5</td>
</tr>
<tr>
<td>Still Life</td>
<td>71</td>
<td>5.8</td>
</tr>
<tr>
<td>Portrait</td>
<td>117</td>
<td>9.6</td>
</tr>
<tr>
<td>Modern Art</td>
<td>110</td>
<td>9.1</td>
</tr>
<tr>
<td>Impressionism</td>
<td>125</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Total: 1215 100.0
Which, if any, of these different types of ... place to eat out would you like the best?  

<table>
<thead>
<tr>
<th>Place Type</th>
<th>$n_k$</th>
<th>$f_k$ (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish &amp; Chips / eat–in restaurant / cafe / teashop</td>
<td>107</td>
<td>8.8</td>
</tr>
<tr>
<td>Pub / Wine bar / Hotel</td>
<td>281</td>
<td>23.1</td>
</tr>
<tr>
<td>Chinese / Thai / Indian Restaurant</td>
<td>402</td>
<td>33.1</td>
</tr>
<tr>
<td>Italian Restaurant / pizza house</td>
<td>228</td>
<td>18.8</td>
</tr>
<tr>
<td>French Restaurant</td>
<td>99</td>
<td>8.1</td>
</tr>
<tr>
<td>Traditional Steakhouse</td>
<td>98</td>
<td>8.1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1215</td>
<td>100.0</td>
</tr>
</tbody>
</table>

$K = 8 + 8 + 7 + 6 = 29$ categories

$n = 1215$ individuals

$8 \times 8 \times 7 \times 6 = 2688$ possible response patterns, only 658 are observed.
Extract from the Individuals x Questions table

<table>
<thead>
<tr>
<th>TV</th>
<th>Film</th>
<th>Art</th>
<th>Eat out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Soap</td>
<td>Action</td>
<td>Landscape</td>
</tr>
<tr>
<td>7</td>
<td>News</td>
<td>Action</td>
<td>Landscape</td>
</tr>
<tr>
<td>31</td>
<td>Soap</td>
<td>Romance</td>
<td>Portrait</td>
</tr>
<tr>
<td>235</td>
<td>News</td>
<td>Drama</td>
<td>Renaissance</td>
</tr>
<tr>
<td>679</td>
<td>Comedy</td>
<td>Horror</td>
<td>Modern</td>
</tr>
<tr>
<td>1215</td>
<td>Soap</td>
<td>Documentary</td>
<td>Landscape</td>
</tr>
</tbody>
</table>

A row corresponds to the *response pattern* of an individual.
### III.4-a. Cloud of Individuals

**Distance** between 2 individuals due to question $q$:

- if $q$ is an **agreement question**:
  - $i$ and $i'$ choose the same category
  - $\Rightarrow$ the distance due to question $q$ is null
  \[ d_q = 0 \]

- if $q$ is a **disagreement question**:
  - $i$ chooses category $k$ and $i'$ chooses category $k'$
  - $\Rightarrow$ the squared distance due to question $q$ is
  \[ d_q^2 = \frac{1}{f_k} + \frac{1}{f_{k'}} \]

The squared overall distance is the mean of the squared distances due to active questions
\[ d^2 = \sum d_q^2 / Q \]
individual $i \rightarrow$ point $M^i$ with relative weight $p_i = \frac{1}{n}$

G: mean point (center) of the cloud

Distance of an individual to the center of the cloud

$$(GM^i)^2 = \left( \frac{1}{Q} \sum_{k \in K_i} \frac{1}{f_k} \right) - 1 \quad (K_i: \text{response pattern of individual } i).$$

Variance of the cloud of individuals

$$V_{\text{cloud}} = \frac{K}{Q} - 1$$

(average number of categories per question minus 1).
III.4-b. Cloud of Categories

**Distance** between categories $k$ and $k'$: 
\[ d^2(k, k') = \frac{n_k + n_{k'} - 2n_{kk'}}{n_k \cdot n_{k'}/n} \]

Category $k \rightarrow$ category–point $M^k$ with relative weight $p_k = f_k/Q$

**G:** mean point (center) of the cloud

**Property**

G is the mean point of the category–points of any question.
Distance of a category–point to the center of the cloud

\[(GM^k)^2 = \frac{1}{f_k} - 1\]

Variance of the cloud of individuals

\[V_{\text{cloud}} = \frac{K}{Q} - 1\]

Contributions

Contribution of category \(k\)
\[\text{Ctr}_k = \frac{1-f_k}{K-Q}\]

Contribution of question \(q\)
\[\text{Ctr}_q = \frac{K_q-1}{K-Q}\]
III.5. Principal Clouds

— *Principal axes*

**Fundamental properties**

- The two clouds have the same variances (eigenvalues).
- \( \sum \lambda = V_{\text{cloud}}, \text{with} \, \bar{\lambda} = \frac{V_{\text{cloud}}}{L} = \frac{1}{Q}. \)

— *Variance rates and modified rates*

Variance rate:

\[
\tau = \frac{\lambda}{V_{\text{cloud}}}
\]

Modified rates = \( \frac{(\lambda - \bar{\lambda})^2}{\sum (\lambda - \bar{\lambda})^2} \) (the sum is over \( \lambda \) such that \( \lambda \geq \bar{\lambda} \))
— Principal coordinates and principal variables

\( y^i_\ell \): coordinate of individual \( i \) on axis \( \ell \)
\[
y^i_\ell = (y^i_\ell)_{i \in I} : \ell\text{-th principal variable over } I
\]

\( y^k_\ell \): coordinate of category \( k \) on axis \( \ell \)
\[
y^K_\ell = (y^K_\ell)_{k \in K} : \ell\text{-th principal variable over } K
\]

Properties

Mean of principal variable is null:
\[
\sum \frac{1}{n} y^i_\ell = 0 \text{ and } \sum p_k y^K_\ell = 0
\]

Variance of principal variable \( \ell \) is equal to \( \lambda_\ell \):
\[
\sum \frac{1}{n} (y^i_\ell)^2 = \lambda_\ell \text{ and } \sum p_k (y^K_\ell)^2 = \lambda_\ell
\]

Principal variables are pairwise uncorrelated:
\[
\ell \neq \ell' \quad \sum y^i_\ell y^j_{\ell'} = 0 \quad \sum y^K_\ell y^K_{\ell'} = 0
\]
III.6. Aids to Interpretation: Contributions

Contribution of category-point $k$ to axis $\ell$: $\frac{py^2}{\lambda}$

($y$: coordinate of point on axis; $p$: relative weight; $\lambda$: variance of axis)

By grouping, contributions add up $\rightarrow$ contribution of question...
The quality of representation of point $M^k$ on axis $\ell$ is

$$\cos^2 \theta_{k\ell} = \frac{(GM^k_{\ell})^2}{(GM^k)^2} = \frac{(y^k_{\ell})^2}{(GM^k)^2}$$
III.7. MCA of the Taste Example

Data set

The data involve:

- \( Q = 4 \) active variables
- \( K = 8 + 8 + 7 + 6 = 29 \) categories
- \( n = 1215 \) individuals

Overall variance of the cloud: \( V_{\text{cloud}} = \frac{29}{4} - 1 = 6.25 \)

Contributions of questions to the overall variance:

\[
\frac{8-1}{29-4} = 28\% \quad 28\% \quad 24\% \quad 20\%
\]
## Elementary statistical results

$8 \times 8 \times 7 \times 6 = 2688$ possible response patterns; 658 are observed.

<table>
<thead>
<tr>
<th>TV</th>
<th>$n_k$</th>
<th>$f_k$</th>
<th>Ctr$_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>News</td>
<td>220</td>
<td>18.1</td>
<td>3.3</td>
</tr>
<tr>
<td>Comedy</td>
<td>152</td>
<td>12.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Police</td>
<td>82</td>
<td>6.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Nature</td>
<td>159</td>
<td>13.1</td>
<td>3.5</td>
</tr>
<tr>
<td>Sport</td>
<td>136</td>
<td>11.2</td>
<td>3.6</td>
</tr>
<tr>
<td>Film</td>
<td>117</td>
<td>9.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Drama</td>
<td>134</td>
<td>11.0</td>
<td>3.6</td>
</tr>
<tr>
<td>Soap operas</td>
<td>215</td>
<td>17.7</td>
<td>3.3</td>
</tr>
<tr>
<td><strong>Films</strong></td>
<td><strong>1215</strong></td>
<td><strong>100.0</strong></td>
<td><strong>28.0</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Art</th>
<th>$n_k$</th>
<th>$f_k$</th>
<th>Ctr$_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>105</td>
<td>8.6</td>
<td>3.7</td>
</tr>
<tr>
<td>Landscape</td>
<td>632</td>
<td>52.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Renaissance</td>
<td>55</td>
<td>4.5</td>
<td>3.8</td>
</tr>
<tr>
<td>Still Life</td>
<td>71</td>
<td>5.8</td>
<td>3.8</td>
</tr>
<tr>
<td>Portrait</td>
<td>117</td>
<td>9.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Modern Art</td>
<td>110</td>
<td>9.1</td>
<td>3.6</td>
</tr>
<tr>
<td>Impressionism</td>
<td>125</td>
<td>10.3</td>
<td>3.6</td>
</tr>
<tr>
<td><strong>Eat out</strong></td>
<td><strong>1215</strong></td>
<td><strong>100.0</strong></td>
<td><strong>24.0</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Eat out</th>
<th>$n_k$</th>
<th>$f_k$</th>
<th>Ctr$_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish &amp; Chips</td>
<td>107</td>
<td>8.8</td>
<td>3.6</td>
</tr>
<tr>
<td>Pub</td>
<td>281</td>
<td>23.1</td>
<td>3.1</td>
</tr>
<tr>
<td>Indian Rest</td>
<td>402</td>
<td>33.1</td>
<td>2.7</td>
</tr>
<tr>
<td>Italian Rest</td>
<td>228</td>
<td>18.8</td>
<td>3.2</td>
</tr>
<tr>
<td>French Rest</td>
<td>99</td>
<td>8.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Steakhouse</td>
<td>98</td>
<td>8.1</td>
<td>3.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1215</strong></td>
<td><strong>100.0</strong></td>
<td><strong>20.0</strong></td>
</tr>
</tbody>
</table>
Basic results of MCA

Dimensionality of the cloud \( \leq K - Q = 29 - 4 = 25 \).

Mean of the variances of axes: \( \frac{\sum \lambda}{25} = 0.25 \).

Axes whose variances exceed the mean.

<table>
<thead>
<tr>
<th>Axes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{variances (}\lambda\text{)})</td>
<td>.400</td>
<td>.351</td>
<td>.325</td>
<td>.308</td>
<td>.299</td>
<td>.288</td>
<td>.278</td>
<td>.274</td>
<td>.268</td>
<td>.260</td>
<td>.258</td>
<td>.251</td>
</tr>
<tr>
<td>(\text{variance rates})</td>
<td>.064</td>
<td>.056</td>
<td>.052</td>
<td>.049</td>
<td>.048</td>
<td>.046</td>
<td>.045</td>
<td>.044</td>
<td>.043</td>
<td>.042</td>
<td>.041</td>
<td>.040</td>
</tr>
<tr>
<td>(\text{modified rates})</td>
<td>.476</td>
<td>.215</td>
<td>.118</td>
<td>.071</td>
<td>.050</td>
<td>.030</td>
<td>.017</td>
<td>.012</td>
<td>.007</td>
<td>.002</td>
<td>.001</td>
<td>.000</td>
</tr>
</tbody>
</table>
Principal coordinates and contributions (in %) of 6 individuals

<table>
<thead>
<tr>
<th>Coordinates</th>
<th>Contributions (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axis 1</td>
</tr>
<tr>
<td>1</td>
<td>+0.135</td>
</tr>
<tr>
<td>7</td>
<td>−0.266</td>
</tr>
<tr>
<td>31</td>
<td>+1.258</td>
</tr>
<tr>
<td>235</td>
<td>−1.785</td>
</tr>
<tr>
<td>679</td>
<td>+1.316</td>
</tr>
<tr>
<td>1215</td>
<td>−0.241</td>
</tr>
</tbody>
</table>
### III – Multiple Correspondence Analysis

**MCA of the Taste Example**

**Relative weight, principal coordinates and contributions (in %) of categories**

<table>
<thead>
<tr>
<th>Television</th>
<th>$p_k$</th>
<th>Axe 1</th>
<th>Axe 2</th>
<th>Axe 3</th>
<th>Axe1</th>
<th>Axe 2</th>
<th>Axe 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV-News</td>
<td>.0453</td>
<td>-0.881</td>
<td>-0.003</td>
<td>-0.087</td>
<td>8.8</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>TV-Comedy</td>
<td>.0313</td>
<td>+0.788</td>
<td>-0.960</td>
<td>-0.255</td>
<td>4.9</td>
<td>8.2</td>
<td>0.6</td>
</tr>
<tr>
<td>TV-Police</td>
<td>.0169</td>
<td>+0.192</td>
<td>+0.405</td>
<td>+0.406</td>
<td>0.2</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>TV-Nature</td>
<td>.0327</td>
<td>-0.775</td>
<td>-0.099</td>
<td>+0.234</td>
<td>4.9</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>TV-Sport</td>
<td>.0280</td>
<td>-0.045</td>
<td>-0.133</td>
<td>+1.469</td>
<td>0.0</td>
<td>0.1</td>
<td>18.6</td>
</tr>
<tr>
<td>TV-Film</td>
<td>.0241</td>
<td>+0.574</td>
<td>-0.694</td>
<td>+0.606</td>
<td>2.0</td>
<td>3.3</td>
<td>2.7</td>
</tr>
<tr>
<td>TV-Drama</td>
<td>.0276</td>
<td>-0.496</td>
<td>-0.053</td>
<td>-0.981</td>
<td>1.7</td>
<td>0.0</td>
<td>8.2</td>
</tr>
<tr>
<td>TV-Soap</td>
<td>.0442</td>
<td>+0.870</td>
<td>+1.095</td>
<td>-0.707</td>
<td>8.4</td>
<td>15.1</td>
<td>6.8</td>
</tr>
</tbody>
</table>

**Film**

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>30.7</td>
<td>27.7</td>
<td>38.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comedy</td>
<td>6.8</td>
<td>1.3</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CostumeDrama</td>
<td>12.7</td>
<td>0.0</td>
<td>13.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Documentary</td>
<td>5.4</td>
<td>0.2</td>
<td>1.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horror</td>
<td>3.8</td>
<td>3.6</td>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Musical</td>
<td>5.5</td>
<td>9.1</td>
<td>9.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Romance</td>
<td>0.2</td>
<td>2.7</td>
<td>2.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Art**

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PerformanceArt</td>
<td>3.0</td>
<td>1.8</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Landscape</td>
<td>1.7</td>
<td>5.6</td>
<td>3.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RenaissanceArt</td>
<td>1.2</td>
<td>0.9</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>StillLife</td>
<td>6.3</td>
<td>2.1</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portrait</td>
<td>5.0</td>
<td>5.9</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ModernArt</td>
<td>2.0</td>
<td>7.1</td>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Eat out**

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish&amp;Chips</td>
<td>0.4</td>
<td>3.9</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pub</td>
<td>1.2</td>
<td>6.5</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IndianRest</td>
<td>5.3</td>
<td>4.0</td>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ItalianRest</td>
<td>0.0</td>
<td>3.9</td>
<td>2.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FrenchRest</td>
<td>8.2</td>
<td>1.4</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steakhouse</td>
<td>0.3</td>
<td>3.5</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15.3</td>
<td>23.1</td>
<td>10.9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

Brigitte Le Roux  (MAP5/CEVIPOF)  
Geometric Data Analysis (GDA)  
Visby, 09/17-21/2018  
60 / 140
Cloud of categories in plane 1-2
Cloud of individuals in plane 1-2.
III.8. Interpretation of Axes

How many axes need to be interpreted?

Axis 1: \( \frac{\lambda_1 - \lambda_2}{\lambda_1} = 0.12 \); modified rate = 0.48
Axis 2: \( \frac{\lambda_2 - \lambda_3}{\lambda_2} = 0.07 \); modified rate = 0.22.
Cumulated modified rate for axes 1 and 2 = 0.70.
After axis 4, variances decrease regularly and the differences are small.

Cumulated modified rate for axes 1, 2 and 3 = 82%
Guide for interpreting an axis

Interpreting an axis amounts to finding out what is similar, on the one hand, between all the elements figuring on the right of the origin and, on the other hand between all that is written on the left; and expressing with conciseness and precision, the contrast (or opposition) between the two extremes.

Benzécri (1992, p. 405)

For interpreting an axis, we use the method of contributions of points and deviations.
Baseline criterion = average contribution = \( \frac{100}{29} \rightarrow 3.4\% \)

The interpretation of an axis is based on the categories whose contributions to axis exceed the average contribution.
Interpretation of axis 1

- **TV (31%)**
  - Left: TV-News 8.8
  - Right: TV-Soap 8.4
  - Right: TV-Nature 4.9
  - Right: TV-Comedy 4.9

- **Film (35%)**
  - Romance 6.8
  - Comedy 6.8
  - Romance 5.5
  - Documentary 5.4
  - Horror 3.8

- **Art (19%)**
  - Portrait 6.3
  - Modern 5.0
  - Renaissance 3.0

- **Eat out (15%)**
  - French Rest. 8.2
  - Indian Rest. 5.3

Total: 43.0 + 46.0 = 89.0
14 categories selected for the interpretation of axis 1: sum of contributions = 89% → good summary

- Axis 1 opposes *matter–of–fact* (and traditional) tastes to *fiction world* (and modern) tastes.
- Axis 2 opposes *popular* to *sophisticated* tastes.
- Axis 3 opposes *outward dispositions* to *inward ones.*
III.9. Transition formulas

Transition formulas express the *relation* between
the *cloud of categories*
and
the *cloud of individuals*.

— **Category mean points**

\[
\overline{M}_k^\ell : \text{category mean point for } k \text{ with coordinate on axis } \ell
\]

\[
\overline{y}_\ell^k = \sqrt{\lambda_\ell} y_\ell^k \quad (\text{second transition formula})
\]

The $K$ category mean points of question $q$ define the
between-$q$ cloud.
• **First transition formula**

\[ \text{category mean point } (\bar{y}^k) \rightarrow \text{category point } (y^k = \frac{1}{\sqrt{\lambda}} \bar{y}^k) \]

*Category–point k is located at the equibarycenter of the \( n_k \) individuals who have chosen category k, up to a stretching along principal axes.*
• Second transition formula

mean for individual $i$ ($\overline{y}^i = \sum_{k \in K_i} y^k / Q$) $\longrightarrow$ individual point $y^i = \frac{1}{\sqrt{\lambda}} \overline{y}^i$

Individual–point is located at the equibarycenter of the $Q$ category–points of his response pattern, up to a stretching along principal axes.
In terms of coordinates:

1. mean of the 4 coordinates on axis 1:

\[
\frac{-0.881 - 1.328 - 1.038 - 1.270}{4} = -1.12925
\]

mean of the 4 coordinates on axis 2:

\[
\frac{-0.003 - 0.037 - 0.747 - 0.488}{4} = -0.31875
\]

2. dividing the coordinate on axis 1 by \(\sqrt{\lambda_1}\):

\[
y_1^i = \frac{-1.12925}{\sqrt{0.4004}} = -1.785
\]

dividing the coordinate on axis 2 by \(\sqrt{\lambda_2}\)

\[
y_2^i = \frac{-0.31875}{\sqrt{0.3512}} = -0.538
\]

which are the coordinates of the individual–point #235.
Supplementary individuals

Plane 1-2. Cloud of 38 Indian immigrants with its mean point (★).
LOCATE YOURSELF
III.10 Supplementary variables

<table>
<thead>
<tr>
<th>weight</th>
<th>Axis 1</th>
<th>Axis 2</th>
<th>Axis 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>513</td>
<td>−0.178</td>
<td>−0.266</td>
</tr>
<tr>
<td>Women</td>
<td>702</td>
<td>+0.130</td>
<td>+0.195</td>
</tr>
<tr>
<td>18-24</td>
<td>93</td>
<td>+0.931</td>
<td>−0.561</td>
</tr>
<tr>
<td>25-34</td>
<td>248</td>
<td>+0.430</td>
<td>−0.322</td>
</tr>
<tr>
<td>35-44</td>
<td>258</td>
<td>+0.141</td>
<td>−0.090</td>
</tr>
<tr>
<td>45-54</td>
<td>191</td>
<td>−0.085</td>
<td>−0.118</td>
</tr>
<tr>
<td>55-64</td>
<td>183</td>
<td>−0.580</td>
<td>+0.171</td>
</tr>
<tr>
<td>≥ 65</td>
<td>242</td>
<td>−0.443</td>
<td>+0.605</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income</th>
<th>weight</th>
<th>Axis 1</th>
<th>Axis 2</th>
<th>Axis 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; $9 000</td>
<td>231</td>
<td>+0.190</td>
<td>+0.272</td>
<td>+0.075</td>
</tr>
<tr>
<td>$10-19 000</td>
<td>251</td>
<td>−0.020</td>
<td>+0.157</td>
<td>−0.004</td>
</tr>
<tr>
<td>$20-29 000</td>
<td>200</td>
<td>−0.038</td>
<td>−0.076</td>
<td>+0.003</td>
</tr>
<tr>
<td>$30-39 000</td>
<td>122</td>
<td>−0.007</td>
<td>−0.071</td>
<td>−0.128</td>
</tr>
<tr>
<td>$40-59 000</td>
<td>127</td>
<td>+0.017</td>
<td>−0.363</td>
<td>+0.070</td>
</tr>
<tr>
<td>&gt; $60 000</td>
<td>122</td>
<td>−0.142</td>
<td>−0.395</td>
<td>−0.018</td>
</tr>
<tr>
<td>“unknown”</td>
<td>162</td>
<td>−0.092</td>
<td>+0.097</td>
<td>−0.050</td>
</tr>
</tbody>
</table>

As a rule of thumb:
— a deviation greater than 0.5 will be deemed to be “notable”;
— a deviation greater than 1, definitely “large”. 
Supplementary questions in plane 1-2 (top), and in plane 2-3 (bottom) (cloud of categories).
— Specific MCA
and
Class Specific Analysis (CSA)

This text is adapted from Chapter 3 (§3.3) of the monograph
Multiple Correspondence Analysis
(QASS series n°163, SAGE, 2010)
Introduction

- **Specific MCA (SpeMCA)** consists in restricting the analysis to categories of interest.

- **Class Specific Analysis (CSA)** consists in analyzing a subset of individuals by taking the whole set of individuals as a reference.
III.11. Specific MCA

The active categories are the *categories of interest*. The excluded categories, called *passive categories*, are:

- **Junk categories**: categories of *no-interest*
  - not representable by a single point

- **Infrequent categories**
  - remote from the center
  - contributing too much to the variance of the question
  - too influential on the determination of axes
Cloud of individuals

If for active question \( q \),
- both \( i \) and \( i' \) choose active categories \( k \) and \( k' \): the distance is unchanged:

\[
d_q^{2'} = \frac{1}{f_k} + \frac{1}{f_{k'}}
\]

- \( i \) chooses active category \( k \) and \( i' \) passive category \( k' \):

\[
d_q^2(i, i') = \frac{1}{f_k} \text{ (dropping } \frac{1}{f_{k'}}\text{ )}
\]

**Geometric viewpoint:**

→ projection of the cloud onto a subspace of interest.
Cloud of categories

subcloud of categories of active questions with weights and distances unchanged.

- **Dimension of the cloud:** $K' - Q'$
  number of active categories ($K'$) minus number of questions without passive categories ($Q'$).

- **Specific overall variance:**
  \[
  \frac{K'}{Q} - \sum_{k \in K'} p_k = \text{sum of eigenvalues}
  \]

- **Modified rates:**
  calculate $\bar{\lambda} = \text{specific variance divided by the number of dimensions of the cloud};$
  modified rates $= \frac{(\lambda - \bar{\lambda})^2}{\sum(\lambda - \bar{\lambda})^2} \left( \sum \text{ over } \lambda > \bar{\lambda} \right).$
Principal axes and principal variables

• Coordinates of individuals on an axis:
  Mean = 0  Variance = specific eigenvalue

• Coordinate of categories on an axis:
  • Mean of coordinates of *active and passive* categories (weighted by the relative weight \( f_k/Q \)) = 0
  • Raw sum of squares of coordinates of *active* categories (weighted by \( p_k = f_k/Q \)) = \( \lambda \)
Fundamental properties of standard MCA are preserved:

- the principal axes of the cloud of individuals are in a one-one correspondence with those of the cloud of categories,

- the two clouds have the same eigenvalues.

Link between the two clouds (transition formulas):

\[ \bar{y} = \sqrt{\lambda} y \]

(y: principal coordinate of category k
\bar{y}: principal coordinate of category mean–point k)
III.12-a. Concentration Ellipses

*geometric summary of a subcloud* in a principal plane

\( \nu_1 = \text{variance of the coordinates of the subcloud on axis 1.} \)

\( \nu_2 = \text{variance of the coordinates of the subcloud on axis 2.} \)

\( c = \text{covariance between the 2 sets of coordinates.} \)

\[
\frac{\nu_2(y_1 - m_1)^2 - 2c(y_1 - m_1)(y_2 - m_2) + \nu_1(y_2 - m_2)^2}{\nu_1 \nu_2 - c^2} = \kappa^2
\]
Properties
The concentration ellipse* of a subcloud is such that the half–axis of the ellipse is along the principal direction of the subcloud projected in the plane under study and its length is equal to $2 \sqrt{\lambda'}$.

• A uniform distribution over the interior of the ellipse has the same variance as the subcloud.
• For a normally–shaped cloud, the concentration ellipse contains about 86% of the points of the cloud.

Concentration ellipses are especially useful for studying families of subclouds induced by a structuring factor or a clustering procedure.

*see Cramér, 1946, p. 284; Le Roux & Rouanet (2010), p.69-T0
From the principal coordinates of the cloud of 10 points coordinates of the mean point C of the subcloud $C$ ($m_1 = +3.8333$, $m_2 = -1.2778$),
variances : $\nu_1 = 25.30612$, $\nu_2 = 21.22449$),
covariance : $c = +7.75510$
Eigenvalues of the covariance matrix: $\gamma_1^2 = 5.59$ and $\gamma_2 = 3.90$;
$tan \alpha_1 = \frac{\gamma_1^2 - \nu_1}{c} = 0.7709$, hence $\alpha_1 = 37.63^\circ$. 

-axis 1 $\lambda_1 = 56$
-axis 2 $\lambda_2 = 36$

G
III.13-b. Class Specific Analysis (CSA)

Study of a class (subset) of individuals with reference to the whole set of individuals.

We seek to

- determine the specific features of the class,
- compare the class subcloud with the initial cloud.
Class specific cloud of individuals

The distance between 2 individuals of the class is the one defined from the whole cloud.
Class specific cloud of categories

The distance between two categories points depends on

- the relative frequencies of the categories in the class,
- the relative frequencies of the categories in the whole set,
- the conjoint frequency of the pairs of categories in the class.
Principal axes and principal variables

• Coordinates of individuals on an axis:
  Mean = 0 \quad \text{Var} = \text{specific eigenvalue}

• Coordinate of categories on an axis (weighted by the relative weight in the whole set):
  Mean = 0 \quad \text{Var} = \text{specific eigenvalue}